Automatica 102 (2019) 129–136

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Robust cooperative output regulation of multi-agent systems via adaptive event-triggered control[☆]

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ARTICLE INFO

Article history:
Received 30 December 2016
Received in revised form 16 July 2018
Accepted 2 January 2019
Available online 6 February 2019

Keywords:
Robust output regulation
Cooperative control
Multi-agent systems
Event-triggered control
Adaptive control

ABSTRACT

This paper investigates the robust cooperative output regulation problem of uncertain linear multi-agent systems with additive disturbances via the celebrated internal model principle. Two novel distributed controllers are designed based on the adaptive control strategy and the event-triggered transmission scheme, where the adaptive control strategy is utilized to avoid the requirement for a priori knowledge of the minimal nonzero eigenvalue of the Laplacian matrix associated with the communication topology, and the event-triggered transmission scheme is utilized to reduce the frequency of data transmission. It is shown that the proposed control schemes achieve robust cooperative output regulation asymptotically and Zeno behavior is excluded. Finally, a simulation example is presented to illustrate the effectiveness of the proposed controllers.

1. Introduction

Recent years have witnessed rapid development in cooperative control of multi-agent systems due to their wide applications in flight formation control (Abdessameud & Tayebi, 2011), multi-vehicle coordination (Ren & Atkins, 2007), power balancing in micro-grids (Cai, Hu, Lewis, & Davoudi, 2016; Yan, Zhou, Zhang, Yang, & Wu, 2017) and so on. As a basic cooperative control problem, consensus has been studied extensively over the past decade. Numerous results have been obtained, see, for example, Ding, Han, Ge, and Zhang (2018), Fax and Murray (2004), Guo, Ding, and Han (2014), Li, Wen, Duan, and Ren (2015), Olfati-Saber and Murray (2004), Scardovi and Sepulchre (2009), Sun and Wang (2009), Yu, Ren, Zheng, Chen, and Lü (2013) and Zhang, Zhou, Wang, Yan, and Sun (2018) and references therein. It should be noted that these works focus on the consensus problem with continuous communication among neighboring agents, which will lead to large communication load and energy consumption. To reduce the communication load and thus save power, the event-triggered transmission scheme has been suggested for distributed control of multi-agent systems. Distributed event-triggered control strategies are proposed in Dimarogonas, Frazzoli, and Johansson (2012), Fan, Feng, Wang, and Song (2013), Fan, Liu, Feng, and Wang (2015), Kia, Cortés, and Martínez (2015), Nowzari and Cortés (2016), Seyboth, Dimarogonas, and Johansson (2013), and Yan, Zhang, Yang, Zhan, and Peng (2018) for single-integrator multi-agent systems. Yang, Ren, Liu, and Chen (2016), Zhang, Feng, Yan, and Chen (2014), and Zhu, Jiang, and Feng (2014) investigate event-triggered consensus for a group of identical agents with general linear dynamics.

More recently, cooperative output regulation, which includes consensus, formation and synchronization as its special cases, has received increasing attention since it can be used to deal with multi-agent systems with uncertainties and disturbances (Hong, Wang, & Jiang, 2013; Li, Feng, Wang, Luo, & Guan, 2014; Seyboth, Ren, & Allgöwer, 2016; Su & Huang, 2012). The objective of cooperative output regulation is to achieve simultaneous asymptotic tracking for the reference input and rejection of the disturbance from the environment. Cooperative output regulation for heterogeneous linear multi-agent systems is addressed in Su and Huang

☆ This work is supported by National Natural Science Foundation of China (61773289, U1713211, 61733013, 61673178), Shanghai International Science and Technology Cooperation Project (18510711100), Shanghai Shuguang Project (16SG28,18SG18), Shanghai Natural Science Foundation (17ZR1444700, 17ZR1445800), Fundamental Research Funds for the Central Universities, and the Research Grants Council of Hong Kong (CityU-11204315). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Wei Ren under the direction of Editor Christos G. Cassandras.

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https://doi.org/10.1016/j.automatica.2019.01.001

(2012), where the control strategy is designed based on a distributed observer. On the basis of Seyboth et al. (2016), Su and Huang (2012) study the cooperative output regulation problem of linear multi-agent systems with global external signal affecting all the agents and local external signals affecting individual agents. Noting that the above works need the knowledge of the eigenvalues of the Laplacian matrix, an adaptive controller is designed in Li et al. (2014) for a group of individual agents with a periodic switching topology. In the aforementioned works, agent matrices are assumed to be perfectly known. However, in practice, there often exist uncertainties in agent matrices due to various reasons. In this case, robust cooperative output regulation has been adopted in Li, Chen, and Ding (2016), Su, Hong, and Huang (2013), Wang, Hong, Huang, and Jiang (2010). The distributed robust cooperative output regulation problem is addressed in Wang et al. (2010) for multi-agent systems where the communication topology is assumed to contain no loop and node 0 is globally reachable. Noting that no cycle assumption is restrictive because it rules out the bidirected graph, a novel distributed controller is developed in Su et al. (2013) for uncertain linear multi-agent systems without such an assumption. A distributed adaptive controller, which needs no knowledge of the eigenvalues of the Laplacian matrix, is proposed in Li et al. (2016) for uncertain linear multi-agent systems with directed topology. In order to reduce the data transmission and inspired by the results of event-triggered consensus, event-triggered cooperative output regulation has been investigated in Hu, Liu, and Feng (2016, 2018), Liu and Huang (2017) and Yang, Zhang, Feng, and Yan (2018). Hu et al. (2016) study the robust cooperative output regulation problem by an event-triggered control strategy based on the combinational measurement approach. A distributed event-triggered controller based on output-feedback is proposed in Liu and Huang (2017) for linear minimum-phase uncertain multi-agent systems.

Although many advanced results on cooperative output regulation have been reported in literature, there are some challenging issues to be addressed. In most of the existing results based on the event-triggered transmission scheme (Hu et al., 2016, 2018; Liu & Huang, 2017), the minimal nonzero eigenvalue of the Laplacian matrix is required when designing control strategies, which is difficult to obtain for large-scale networks. Besides, uncertain agent matrices will bring significant difficulties when designing event-triggered control strategies. Thus, event-triggered robust cooperative output regulation for uncertain linear multi-agent systems is still a challenging task. All these observations motivate the current study.

In this paper, robust cooperative output regulation is considered for heterogeneous linear multi-agent systems with parameter uncertainties and additive disturbances. The main contributions can be summarized as follows. 1. Two novel adaptive event-triggered control strategies based on the internal model principle are proposed for solving the robust cooperative output regulation problem of the uncertain linear multi-agent systems with and without additive disturbances respectively. 2. The proposed adaptive event-triggered controllers are fully distributed in the sense that they need no global knowledge of the system such as the minimal nonzero eigenvalue of the Laplacian matrix. 3. A novel event-triggering mechanism with adaptive threshold parameters is proposed. With this mechanism, data transmission will be greatly reduced and Zeno behavior is also excluded.

2. Robust cooperative output regulation

2.1. Preliminaries and problem formulation

Consider a group of N individual agents with uncertain linear dynamics described by

$$\begin{aligned}\dot{x}_i(t) =& A_{\omega i}x_i(t) + B_{\omega i}u_i(t) + E^d_{\omega i}\varpi(t),\\ y_i(t) =& C_{\omega i}x_i(t),\ i = 1, \dots, N,\end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the state, $u_i(t) \in \mathbb{R}^{m_i}$ the control input, and $y_i(t) \in \mathbb{R}^p$ the output of agent i respectively, $\varpi(t) \in \mathbb{R}^{q_w}$ is the environment disturbance. The system matrices $A_{\omega i} \in \mathbb{R}^{n_i\times n_i}$, $B_{\omega i} \in \mathbb{R}^{n_i\times m_i}$, $E^d_{\omega i} \in \mathbb{R}^{n_i\times q_w}$ and $C_{\omega i} \in \mathbb{R}^{p\times n_i}$ are uncertain matrices in the following form, $A_{\omega i} = A_i+\Delta A_i$, $B_{\omega i} = B_i+\Delta B_i$, $C_{\omega i} = C_i+\Delta C_i$, $E^d_{\omega i} = E^d_i + \Delta E^d_i$. Denote $v(t) = [r^\top(t)\ \varpi^\top(t)]^\top \in \mathbb{R}^q$, where $r(t) \in \mathbb{R}^{q-q_w}$ is the reference input. Eq. (1) can be rewritten in the form

$$\begin{aligned}&\dot{x}_i(t) = A_{\omega i}x_i(t) + B_{\omega i}u_i(t) + E_{\omega i}v(t),\\ &y_i(t) = C_{\omega i}x_i(t),\ i = 1, \dots, N,\end{aligned} \tag{2}$$

where $E_{\omega i} = E_i + \Delta E_i$, $E_i = [0\ E^d_i] \in \mathbb{R}^{n_i\times q}$, and $\Delta E_i = [0\ \Delta E^d_i] \in \mathbb{R}^{n_i\times q}$. The external signal $v(t)$ composed of the reference input $r(t)$ and the environment disturbance $\varpi(t)$ is generated by the following exosystem

$$\begin{aligned}\dot{v}(t) =& Sv(t),\\ y_0(t) =& -\bar{F}r(t) = -Fv(t),\end{aligned} \tag{3}$$

where $y_0(t) \in \mathbb{R}^p$ is the output of the exosystem, $S \in \mathbb{R}^{q\times q}$, $\bar{F} \in \mathbb{R}^{p\times(q-q_w)}$, and $F = [\bar{F}\ 0] \in \mathbb{R}^{p\times q}$. The regulated output is defined as $e_i(t) = y_i(t) - y_0(t)$, which yields

$$e_i(t) = C_{\omega i}x_i(t) + Fv(t). \tag{4}$$

Assume that A_i, B_i, C_i, E_i, F and S are known matrices, and $\Delta A_i, \Delta B_i, \Delta C_i, \Delta E_i$ are perturbations of the corresponding matrices respectively. For convenience, define

$$\omega = \begin{bmatrix}\mathrm{vec}(\Delta A_1, \Delta B_1, \Delta E_1)\\ \cdots\\ \mathrm{vec}(\Delta A_N, \Delta B_N, \Delta E_N)\\ \mathrm{vec}(\Delta C_1, \dots, \Delta C_N)\end{bmatrix} \in \mathbb{R}^{\sum_{i=1}^N n_i(n_i+m_i+p+q)}.$$

The system with $\omega = 0$ is called a nominal system.

Consider a graph $\mathcal{G}$ associated with the system consisting of N agents and one exosystem. The communication topology among N agents is described by an undirected graph $\mathcal{G}_s = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, \dots, N\}$ and $\mathcal{E} \in \mathcal{N}\times\mathcal{N}$ are the node set and the edge set of graph $\mathcal{G}_s$ respectively. An edge $(i, j) \in \mathcal{E}$ represents the communication channel from node i to node j. The graph $\mathcal{G}_s$ is undirected if $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$. The neighbor set of agent i is denoted by $\mathcal{N}_i = \{j \in \mathcal{N} \mid (j, i) \in \mathcal{E}\}$. $\mathcal{A}_s = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix of $\mathcal{G}_s$, and $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$; $a_{ij} = 0$ otherwise. $\mathcal{D}_s = \mathrm{diag}\{\sum_{j\in\mathcal{N}_1} a_{1j}, \dots, \sum_{j\in\mathcal{N}_N} a_{Nj}\}$ is the degree matrix of $\mathcal{G}_s$, and the Laplacian matrix of $\mathcal{G}_s$ is denoted by $\mathcal{L}_s = \mathcal{D}_s - \mathcal{A}_s$. For the undirected graph, the Laplacian matrix $\mathcal{L}_s = \mathcal{L}_s^\top \geq 0$. $\mathcal{D}_l = \mathrm{diag}\{a_{10}, \dots, a_{N0}\}$ denotes the weights of directed edges from the exosystem (labeled 0) to the agents i, $i = 1, \dots, N$. If $a_{i0} = 1$, agent i can receive the information from the exosystem; $a_{i0} = 0$, otherwise. Node 0 is globally reachable if node 0 has directed paths in $\mathcal{G}$ to all other nodes in $\mathcal{G}_s$. The following assumption is made on $\mathcal{G}$.

Assumption 1. Node 0 is globally reachable in the graph $\mathcal{G}$.

Since the exosystem indexed by 0 has no neighbor, the Laplacian matrix $\mathcal{L}$ of the graph $\mathcal{G}$ can be partitioned as $\mathcal{L} = \begin{bmatrix}0 & 0_{1\times N}\\ \mathcal{H}_1 & \mathcal{H}\end{bmatrix}$, where $\mathcal{H} = \mathcal{L}_s + \mathcal{D}_s$ and $\mathcal{H}_1 = -\mathcal{D}_l\mathbf{1}_N$.

Lemma 1 (Hu & Hong, 2007)**.** *The matrix $\mathcal{H}$ is positive stable if and only if node 0 is globally reachable in graph $\mathcal{G}$.*

Before proceeding further, some assumptions and definitions are introduced next.

Assumption 2. S has no eigenvalues with negative real parts.

Assumption 3. The pair (A_i, B_i) is stabilizable, where A_i and B_i are the system matrices of the nominal system.

Definition 1 (*Huang, 2004*). The pair (G_1, G_2) incorporates the minimal p-copy internal model of S if $G_1 = \mathrm{diag}\{\Upsilon_{11}, \ldots, \Upsilon_{1p}\}$ and $G_2 = \{\Upsilon_{21}, \ldots, \Upsilon_{2p}\}$, where Υ_{1s}, $s = 1, \ldots, p$ are square matrices, and Υ_{2s}, $s = 1, \ldots, p$ are column vectors such that $(\Upsilon_{1s}, \Upsilon_{2s})$ is controllable, and the minimal polynomial of S equals the characteristic polynomial of Υ_{1s}.

Assumption 4. For all $\lambda \in \Lambda(S)$, $\mathrm{rank}\begin{bmatrix} A_i-\lambda I & B_i \\ C_i & 0 \end{bmatrix} = n_i + p$, where $\Lambda(S)$ is the spectrum of S, and S is the system matrix of the exogenous system.

Remark 1. Assumptions 2–4 are standard ones for solving the linear robust output regulation problem by the classical method in Huang (2004). From Lemma 1.26 of Huang (2004), it can be obtained that under Assumptions 2–4, if the pair (G_1, G_2) incorporates a p-copy internal model of S, the pair $\left(\begin{bmatrix} A_i & 0 \\ G_2C_i & G_1 \end{bmatrix}, \begin{bmatrix} B_i \\ 0 \end{bmatrix}\right)$ is stabilizable.

The objective of this section is to design an adaptive event-triggered controller such that the uncertain linear multi-agent system achieves asymptotic reference tracking and disturbance rejection. More specifically, the robust cooperative output regulation problem can be described as follows.

Definition 2 (*Su et al., 2013*). Given multi-agent systems (2) and the exosystem (3), design a control strategy such that

(1) the nominal closed-loop system is asymptotically stable when $v(t) = 0$; and

(2) there exists an open neighborhood $\mathcal{W}$ of $\omega = 0$ such that for any $\omega \in \mathcal{W}$ and any initial condition, $\lim_{t\to\infty} e_i(t) = 0$, $i = 1, \ldots, N$.

2.2. Main results

In this subsection, a novel controller is presented to solve the robust cooperative output regulation problem of the uncertain linear multi-agent systems. For each agent, the sequence of the event-triggering instants is denoted by $t_0^i, t_1^i, \ldots$, where t_k^i is the kth event of agent i and the event is defined by a condition to be designed in (6). If an event is triggered, agent i will broadcast its information to its neighbors. For $t \in [t_k^i, t_{k+1}^i)$, the adaptive control strategy based on the event-triggered transmission scheme is designed as

$$\begin{aligned}
u_i(t) =& K_{xi}x_i(t) + K_{zi}z_i(t), \\
\dot{z}_i(t) =& G_1z_i(t) + G_2(y_i(t) + F\hat{v}_i(t)), \\
\dot{\hat{v}}_i(t) =& S\hat{v}_i(t) - d_i(t)\phi_i(t), \\
\phi_i(t) =& \sum_{j=1}^{N} a_{ij}(\mathrm{e}^{S(t-t_k^i)}\hat{v}_i(t_k^i) - \mathrm{e}^{S(t-t_{k'}^j)}\hat{v}_j(t_{k'}^j)) \\
& + a_{i0}(\mathrm{e}^{S(t-t_k^i)}\hat{v}_i(t_k^i) - \mathrm{e}^{S(t-t_k^i)}v(t_k^i)), \\
\dot{d}_i(t) =& \alpha_i\phi_i^{\top}(t)\phi_i(t),
\end{aligned} \tag{5}$$

where $t_{k'}^j$ is the latest triggering instant of agent j before current t, that is, $k'(t) = \arg\min_l\{t - t_l^j \mid t \geq t_l^j, l \in \mathbb{N}\}$. $z_i(t) \in \mathbb{R}^{n_s p}$ is the state of the dynamic compensator, n_s is the degree of the minimal polynomial of S, $\hat{v}_i(t) \in \mathbb{R}^q$ is the estimate of the external signal $v(t)$, and $d_i(t)$ is the adaptive parameter satisfying $d_i(0) \geq 1$. The pair (G_1, G_2) incorporates the minimal p-copy internal model of S. The scalar $\alpha_i > 0$ is a positive constant. $K_{xi} \in \mathbb{R}^{m_i\times n_i}$ and $K_{zi} \in \mathbb{R}^{m_i\times n_s p}$ are gain matrices to be determined later.

Remark 2. In controller (5), neighbors' signals at their triggering instants $\hat{v}_j(t_{k'}^j)$ and the state transmission matrix $\mathrm{e}^{S(t-t_{k'}^j)}$ are utilized to avoid continuous measurement and transmission of neighboring agent information $\hat{v}_j(t)$. In this way, data transmission can be greatly reduced in comparison with those existing works relying on continuous transmission among agents, such as Li et al. (2016) and Seyboth et al. (2016).

The measurement error for each agent is defined as $e_{\hat{v}i}(t) = \mathrm{e}^{S(t-t_k^i)}\hat{v}_i(t_k^i) - \hat{v}_i(t)$. The event-triggering mechanism invokes transmission of signals when the measurement error becomes larger than the threshold. More specifically, the event-triggering mechanism can be expressed as

$$\begin{aligned}
t_{k+1}^i =& \inf\{t > t_k^i | \psi_i(t)d_i(t)e_{\hat{v}i}^{\top}(t)e_{\hat{v}i}(t) > \mathrm{e}^{-\gamma t}\}, \\
\dot{\psi}_i(t) =& \beta_i d_i(t)e_{\hat{v}i}^{\top}(t)e_{\hat{v}i}(t),
\end{aligned} \tag{6}$$

where $\psi_i(t)$ is the adaptive parameter satisfying $\psi_i(0) \geq 1$. The scalars $\beta_i > 0$ and $\gamma > 0$ are positive constants.

Remark 3. Under the proposed event-triggering mechanism (6), each agent broadcasts its states to neighbors only at its own event instants. Compared with Dimarogonas et al. (2012) and Zhang et al. (2014), continuous communication among neighbors is no longer required.

Substituting (5) into (2), for $t \in [t_k^i, t_{k+1}^i)$, the closed-loop system can be written as

$$\begin{aligned}
\dot{\tilde{x}}_i(t) =& (A_{\omega i} + B_{\omega i}K_{xi})\tilde{x}_i(t) + B_{\omega i}K_{zi}\tilde{z}_i(t), \\
\dot{\tilde{z}}_i(t) =& G_1\tilde{z}_i(t) + G_2(C_{\omega i}\tilde{x}_i(t) + F\bar{v}_i(t)), \\
\dot{\bar{v}}_i(t) =& S\bar{v}_i(t) - d_i(t)\sum_{j=1}^{N} a_{ij}(\bar{v}_i(t) - \bar{v}_j(t) + e_{\hat{v}i}(t) \\
& - e_{\hat{v}j}(t)) - d_i(t)a_{i0}(\bar{v}_i(t) + e_{\hat{v}i}(t)),
\end{aligned} \tag{7}$$

where $\tilde{x}_i(t) = x_i(t) - X_iv(t)$, $\tilde{z}_i(t) = z_i(t) - Z_iv(t)$ and $\bar{v}_i(t) = \hat{v}_i(t) - v(t)$. Here the pair (X_i, Z_i) is the unique solution of

$$\begin{aligned}
X_iS =& (A_{\omega i} + B_{\omega i}K_{xi})X_i + B_{\omega i}K_{zi}Z_i + E_{\omega i}, \\
Z_iS =& G_1Z_i + G_2(C_{\omega i}X_i + F).
\end{aligned} \tag{8}$$

Moreover, X_i also satisfies $0 = C_{\omega i}X_i + F$.

Now, we are ready to present the main results of this section. First, the feasibility of the event-triggered control scheme will be analyzed by excluding Zeno Behavior.

Theorem 1. *Consider the multi-agent system (2) with the exosystem (3). Suppose that Assumptions 1–4 hold. Let the following statements hold*

(a) *Choose $G_1 = diag\{\Upsilon_{11}, \ldots, \Upsilon_{1p}\}$, and $G_2 = diag\{\Upsilon_{21}, \ldots, \Upsilon_{2p}\}$, where*

$$\Upsilon_{1s} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_{n_s} & -a_{n_s-1} & \cdots & -a_1 \end{bmatrix}, \Upsilon_{2s} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, s = 1, \ldots, p,$$

and $f(\lambda) = -\lambda^{n_s} - a_1\lambda^{n_s-1} - \cdots - a_{n_s-1}\lambda - a_{n_s}$ is the minimal polynomial of S.

(b) *Choose $\varrho_1 > 0$ and $\mu_i > 0$ such that there exists a positive definite matrix $P_{1i} \in \mathbb{R}^{(n_i+n_sp)\times(n_i+n_sp)}$ satisfying*

$$P_{1i}A_{ci} + A_{ci}^{\top}P_{1i} - 2\mu_iP_{1i}B_{ci}B_{ci}^{\top}P_{1i} + \varrho_1 I = 0, \tag{9}$$

where $A_{ci} = \begin{bmatrix} A_i & 0 \\ G_2C_i & G_1 \end{bmatrix}$ and $B_{ci} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$.

(c) *Choose $\gamma > 0$, $\alpha_i > 0$, and $\beta_i > 0$. The controller gain matrices are designed as $[K_{xi}\ K_{zi}] = -\mu_i[B_i^{\top}\ 0]P_{1i}$.*

Then, under the adaptive event-triggered controller (5) and the event-triggering scheme (6), no agent will exhibit Zeno behavior.

Proof. From (9) and the definition of K_{xi} and K_{zi}, $\begin{bmatrix} A_i+B_iK_{xi} & B_iK_{zi} \\ G_2C_i & G_1 \end{bmatrix}$ is Hurwitz. Therefore, there exists an open neighborhood $\mathcal{W}$ of $w=0$ such that for each $w \in \mathcal{W}$, $\hat{A}_{ci} = \begin{bmatrix} A_{wi}+B_{wi}K_{xi} & B_{wi}K_{zi} \\ G_2C_{wi} & G_1 \end{bmatrix}$ is Hurwitz. There exists $P_{2i} > 0$ such that $P_{2i}\hat{A}_{ci} + \hat{A}_{ci}^{\top}P_{2i} \leq -(\hat{\varrho}_1 + \epsilon)I$, $\hat{\varrho}_1 > 0$ and $\epsilon > 0$.

Consider the Lyapunov function candidate $V(t) = \sum_{k=1}^{4} V_k(t)$, where $V_1(t) = \sum_{i=1}^{N} x_{ci}^{\top}(t)P_{2i}x_{ci}(t)$, $V_2(t) = \bar{v}^{\top}(t)(\mathcal{H}\otimes I_q)\bar{v}(t)$, $V_3(t) = \sum_{i=1}^{N}\frac{1}{4\alpha_i}\tilde{d}_i^2(t)$, $V_4(t) = \sum_{i=1}^{N}\frac{1}{2\beta_i}\tilde{\psi}_i^2(t)$, $x_{ci}(t) = [\tilde{x}_i^{\top}(t), \tilde{z}_i^{\top}(t)]^{\top}$, $\bar{v}(t) = [\bar{v}_1^{\top}(t), \ldots, \bar{v}_N^{\top}(t)]^{\top}$, $\tilde{d}_i(t) = d_i(t) - a_1$, $\tilde{\psi}_i(t) = \psi_i(t) - a_2$ and scalars a_1 and a_2 are constant parameters to be determined later.

Define a strictly increasing sequence $\{t_m\} = \bigcup_{i=1}^{N}\{t_k^i\}$, $m = 0, 1, \ldots$, $k = 0, 1, \ldots$, which represents the union of all the triggering instants. Then, taking the time derivative of $V_1(t)$ along the trajectories of (7) over $[t_m, t_{m+1})$ yields

$$\begin{aligned} \dot{V}_1(t) \leq & \sum_{i=1}^{N} x_{ci}^{\top}(t)(P_{2i}\hat{A}_{ci} + \hat{A}_{ci}^{\top}P_{2i} + \epsilon I)x_{ci}(t) \\ & + \sum_{i=1}^{N}\frac{1}{\epsilon}\|P_{2i}\|^2\bar{v}_i^{\top}(t)F^{\top}G_2^{T}G_2F\bar{v}_i(t). \end{aligned} \tag{10}$$

Calculating the time derivative of $V_2(t)$ over $[t_m, t_{m+1})$, one has

$$\begin{aligned} \dot{V}_2(t) \leq & \bar{v}^{\top}(t)(\mathcal{H}\otimes(S+S^{\top}) - \mathcal{H}\mathcal{D}(t)\mathcal{H}\otimes I_q)\bar{v}(t) \\ & + e_{\hat{v}}^{\top}(t)(\mathcal{H}\mathcal{D}(t)\mathcal{H}\otimes I_q)e_{\hat{v}}(t), \end{aligned} \tag{11}$$

where $e_{\hat{v}}(t) = [e_{\hat{v}1}^{\top}(t), \ldots, e_{\hat{v}N}^{\top}(t)]^{\top}$ and $\mathcal{D}(t) = \mathrm{diag}\{d_1(t), \ldots, d_N(t)\}$. Similarly, the time derivative of $V_3(t)$ and $V_4(t)$ in $[t_m, t_{m+1})$ are given, respectively, by

$$\begin{aligned} \dot{V}_3(t) \leq & \sum_{i=1}^{N}\frac{1}{2}(d_i(t) - a_1)\phi_i^{\top}(t)\phi_i(t) \\ \leq & \bar{v}^{\top}(t)(\mathcal{H}\mathcal{D}(t)\mathcal{H}\otimes I_q)\bar{v}(t) + e_{\hat{v}}^{\top}(t)(\mathcal{H}\mathcal{D}(t)\mathcal{H} \\ & \otimes I_q)e_{\hat{v}}(t) - \frac{a_1}{4}\bar{v}^{\top}(t)(\mathcal{H}\mathcal{H}\otimes I_q)\bar{v}(t) \\ & + \frac{a_1}{2}e_{\hat{v}}^{\top}(t)(\mathcal{H}\mathcal{H}\otimes I_q)e_{\hat{v}}(t), \end{aligned} \tag{12}$$

and

$$\dot{V}_4(t) = \sum_{i=1}^{N}(\psi_i(t) - a_2)d_i(t)e_{\hat{v}i}^{\top}(t)e_{\hat{v}i}(t). \tag{13}$$

Combining (10)–(13), for $t \in [t_m, t_{m+1})$, one can obtain that

$$\begin{aligned} \dot{V}(t) \leq & -\hat{\varrho}_1\sum_{i=1}^{N}x_{ci}^{\top}(t)x_{ci}(t) + (2\|S\|\|\mathcal{H}\| - \frac{a_1}{4}\lambda_{\min}(\mathcal{H}^{\top}\mathcal{H}) \\ & + \frac{p_2}{\epsilon}\|G_2F\|^2)\bar{v}^{\top}(t)\bar{v}(t) + \sum_{i=1}^{N}4d_i(t)|\mathcal{N}_i|^2 e^{-\gamma t} \\ & + \sum_{i=1}^{N}(4d_i(t)(|\mathcal{N}_i| + a_{i0})^2 + \frac{a_1}{2}\|\mathcal{H}\|^2)e_{\hat{v}i}^{\top}(t)e_{\hat{v}i}(t) \\ & + \sum_{i=1}^{N}(\psi_i(t) - a_2)d_i(t)e_{\hat{v}i}^{\top}(t)e_{\hat{v}i}(t) \\ \leq & -\hat{\varrho}_1\sum_{i=1}^{N}x_{ci}^{\top}(t)x_{ci}(t) - \varrho_2\sum_{i=1}^{N}\bar{v}_i^{\top}(t)\bar{v}_i(t) \\ & + \sum_{i=1}^{N}e^{-\gamma t} + \sum_{i=1}^{N}4d_i(t)N^2e^{-\gamma t}, \end{aligned} \tag{14}$$

where $a_1 \geq \frac{4}{\lambda_{\min}(\mathcal{H}^{\top}\mathcal{H})}(2\|S\|\|\mathcal{H}\| + \frac{p_2\|G_2F\|^2}{\epsilon} + \varrho_2)$, $a_2 \geq \frac{a_1}{2}\|\mathcal{H}\|^2 + 4(N+1)^2$, $p_2 = \max_{i=1,\ldots,N}\|P_{2i}\|^2$, and $\varrho_2 > 0$ will be determined later. Define $W(t) = V(t) + V_5(t)$, where $V_5(t) = \frac{a_3}{\gamma}\sum_{i=1}^{N}e^{-\gamma t} + \frac{1}{\gamma}\sum_{i=1}^{N}4d_i(t)N^2e^{-\gamma t}$, where a_3 is a constant to be determined later. Then, taking the time derivative of $W(t)$ over $[t_m, t_{m+1})$, one has

$$\begin{aligned} \dot{W}(t) \leq & (\frac{8}{\gamma}N^2\alpha_{\max}\|\mathcal{H}\|^2 - \varrho_2)\sum_{i=1}^{N}\bar{v}_i^{\top}(t)\bar{v}_i(t) + (1 - a_3 \\ & + \frac{8}{\gamma}N^2\alpha_{\max}\|\mathcal{H}\|^2)\sum_{i=1}^{N}e^{-\gamma t} - \hat{\varrho}_1\sum_{i=1}^{N}x_{ci}^{\top}(t)x_{ci}(t) \\ \leq & -\hat{\varrho}_1\sum_{i=1}^{N}x_{ci}^{\top}(t)x_{ci}(t) - \varrho_3\sum_{i=1}^{N}\bar{v}_i^{\top}(t)\bar{v}_i(t) \leq 0, \end{aligned} \tag{15}$$

where $a_3 \geq 1 + \frac{8}{\gamma}N^2\alpha_{\max}\|\mathcal{H}\|^2$, $\varrho_2 \geq \frac{8}{\gamma}N^2\alpha_{\max}\|\mathcal{H}\|^2 + \varrho_3$, $\varrho_3 > 0$ and $\alpha_{\max} = \max_{i=1,\ldots,N}\alpha_i$. Therefore, one can obtain that $x_{ci}(t)$, $\bar{v}_i(t)$, $d_i(t)$ and $\psi_i(t)$ are bounded over $[0, t_{\infty})$, where $t_{\infty} = \lim_{m\to\infty}t_m$.

Then, Zeno behavior will be excluded by contradiction. Suppose that Zeno behavior exhibits for agent i, that is, $\lim_{k\to\infty}t_{k+1}^i = T_0 < \infty$. Taking the upper right-hand Dini derivative of $e_{\hat{v}i}^{\top}(t)e_{\hat{v}i}(t)$ over $[t_k^i, t_{k+1}^i)$, one has

$$\begin{aligned} & D^{+}(e_{\hat{v}i}^{\top}(t)e_{\hat{v}i}(t)) \\ \leq & (2\|S\|+1)e_{\hat{v}i}^{\top}(t)e_{\hat{v}i}(t) + d_i^2(t)\phi_i^{\top}(t)\phi_i(t) \\ \leq & (2\|S\|+1)e_{\hat{v}i}^{\top}(t)e_{\hat{v}i}(t) + 2d_i^2(t)\|\mathcal{H}\|^2(\frac{W(0)}{\lambda_{\min}(\mathcal{H})} + N). \end{aligned} \tag{16}$$

Since $d_i(t)$ is monotonically nondecreasing, for $t \in [t_k^i, t_{k+1}^i)$, one can calculate that

$$\begin{aligned} e_{\hat{v}i}^{\top}(t)e_{\hat{v}i}(t) \leq & \frac{2d_i^2(t)\|\mathcal{H}\|^2}{2\|S\|+1}(\frac{W(0)}{\lambda_{\min}(\mathcal{H})} + N) \\ & \times (e^{(2\|S\|+1)(t-t_k^i)} - 1). \end{aligned} \tag{17}$$

Define $\Psi_i(t) = \psi_i(t)d_i(t)e_{\hat{v}i}^{\top}(t)e_{\hat{v}i}(t)$. From (17), one has

$$\begin{aligned} \Psi_i(t) \leq & \frac{2\psi_i(t)d_i^3(t)\|\mathcal{H}\|^2}{2\|S\|+1}(\frac{W(0)}{\lambda_{\min}(\mathcal{H})} + N) \\ & \times (e^{(2\|S\|+1)(t-t_k^i)} - 1),\ t \in [t_k^i, t_{k+1}^i). \end{aligned} \tag{18}$$

Then, according to (6) and (18), one can obtain that the inter-event interval satisfies $t_{k+1}^i - t_k^i \geq \frac{1}{2\|S\|+1}\ln(1 + \frac{e^{-\gamma T_0}(2\|S\|+1)}{2\psi_i(T_0)d_i^3(T_0)\|\mathcal{H}\|^2(\frac{W(0)}{\lambda_{\min}(\mathcal{H})}+N)}) = \tau_i$. Since $d_i(t)$ and $\psi_i(t)$ are monotonically nondecreasing and bounded over $[0, t_{\infty})$, it can be obtained that $\tau_i > 0$. Then, one can obtain that $\lim_{k\to\infty}t_{k+1}^i \geq \lim_{k\to\infty}(k+1)\tau_i = \infty$, which contradicts the hypothesis $\lim_{k\to\infty}t_{k+1}^i = T_0 < \infty$. Therefore, Zeno behavior for agent i can be excluded. In other words, for any given finite time period, there will be only finite number of events for any agent. □

Remark 4. Since Assumptions 2–4 hold and the pair (G_1, G_2) incorporates the minimal p-copy internal model of S, the pair (A_{ci}, B_{ci}) is thus stabilizable. According to Theorem 4.1 in Wonham (1968), $P_{1i} > 0$ is the unique solution of (9) if (A_{ci}, B_{ci}) is stabilizable. Therefore, the pair (A_{ci}, B_{ci}) being stabilizable is a necessary condition for the solvability of (9).

Remark 5. Compared with some existing works on the event-triggered (Hu et al., 2018; Liu & Huang, 2017; Zhang et al., 2014) and the self-triggered (Fan et al., 2015) strategies, the proposed control strategy needs no global knowledge such as the nonzero minimal eigenvalue of the Laplacian matrix and the number of the agents.

Then, the solvability of the robust cooperative output regulation problem will be shown as follows.

Theorem 2. *Consider the multi-agent system (2) and the exosystem (3) with the controller (5) and the event-triggering mechanism (6). Robust cooperative output regulation can be achieved asymptotically for the closed-loop system.*

Proof. According to the proof in Theorem 1, the positive definite function $W(t)$ is decreasing on $[0,\infty)$ and hence $\lim_{t\to\infty} W(t)$ exists. By Cauchy rule, for every $c > 0$, there exists a constant $\varepsilon > 0$, such that when $t'' > t' > \varepsilon$, $W(t') - W(t'') < c$. Therefore,

$$\begin{aligned}
&\int_{t'}^{t''} \sum_{i=1}^{N} (\hat{\varrho}_1 x_{ci}^\top(\tau) x_{ci}(\tau) + \varrho_3 \bar{v}_i^\top(\tau)\bar{v}_i(\tau)) d\tau \\
&= \int_{t'}^{t_{m_1}} \sum_{i=1}^{N} (\hat{\varrho}_1 x_{ci}^\top(\tau) x_{ci}(\tau) + \varrho_3 \bar{v}_i^\top(\tau)\bar{v}_i(\tau)) d\tau \\
&\quad + \int_{t_{m_1}}^{t_{m_2}} \sum_{i=1}^{N} (\hat{\varrho}_1 x_{ci}^\top(\tau) x_{ci}(\tau) + \varrho_3 \bar{v}_i^\top(\tau)\bar{v}_i(\tau)) d\tau \\
&\quad + \cdots + \int_{t_{m_s}}^{t''} \sum_{i=1}^{N} (\hat{\varrho}_1 x_{ci}^\top(\tau) x_{ci}(\tau) + \varrho_3 \bar{v}_i^\top(\tau)\bar{v}_i(\tau)) d\tau \\
&\le W(t') - W(t_{m_1}) + W(t_{m_1}) - W(t_{m_2}) + \cdots \\
&\quad + W(t_{m_s}) - W(t'') \\
&= W(t') - W(t'') < c.
\end{aligned} \tag{19}$$

By Cauchy rule, one can conclude that $\int_0^\infty \sum_{i=1}^N x_{ci}^\top(\tau) \times x_{ci}(\tau)d\tau$ and $\int_0^\infty \sum_{i=1}^N \bar{v}_i^\top(\tau)\bar{v}_i(\tau)d\tau$ are convergent. Since $x_{ci}(t)$, $\bar{v}_i(t)$, $d_i(t)$ and $\psi_i(t)$ are bounded over $[0,\infty)$, from (7), one can obtain that $\sum_{i=1}^N x_{ci}^\top(\tau)\dot{x}_{ci}(\tau)d\tau$ and $\sum_{i=1}^N \bar{v}_i^\top(\tau)\dot{\bar{v}}_i(\tau)d\tau$ are bounded over $[t_m, t_{m+1})$, $m = 0, 1, \ldots$. Therefore, from Lemma 1 in Su and Huang (2012), it can be concluded that $\lim_{t\to\infty} \tilde{x}_i(t) = 0$, $\lim_{t\to\infty} \tilde{z}_i(t) = 0$ and $\lim_{t\to\infty} \bar{v}_i(t) = 0$.

Then, according to the equation $C_{\omega i}X_i + F = 0$, one has $e_i(t) = C_{\omega i}\tilde{x}_i(t) + (C_{\omega i}X_i + F)v(t) = C_{\omega i}\tilde{x}_i(t)$. Since $\lim_{t\to\infty}\tilde{x}_i(t) = 0$, one can obtain that for any $\omega \in \mathcal{W}$ and any initial condition, $\lim_{t\to\infty} e_i(t) = 0$. The proof of the theorem is thus completed. □

3. Robust cooperative output regulation with additive disturbance

In this section, the multi-agent system with additive disturbance is considered. The dynamics of the agents can be written in the following form

$$\begin{aligned}
\dot{x}_i(t) &= A_{\omega i}x_i(t) + B_{\omega i}u_i(t) + E_{\omega i}v(t) + E_{di}\vartheta_i(t), \\
y_i(t) &= C_{\omega i}x_i(t),\ i = 1,\ldots,N,
\end{aligned} \tag{20}$$

where $\vartheta_i(t) \in \mathbb{R}$ is the additive bounded disturbance with its upper bound $\bar{\vartheta}_i > 0$, and the rest of the parameters are defined as in (1). The matrix $E_{di} \in \mathbb{R}^{n_i\times 1}$ describes how the additive disturbance affects the system. For $t \in [t_k^i, t_{k+1}^i)$, the adaptive event-triggered controller in this case is designed as

$$\begin{aligned}
u_i(t) =& K_{xi}x_i(t) + K_{zi}z_i(t), \\
\dot{z}_i(t) =& G_1 z_i(t) + G_2(y_i(t) + F\hat{v}_i(t)), \\
\dot{\hat{v}}_i(t) =& S\hat{v}_i(t) - d_i(t)\phi_i(t), \\
\phi_i(t) =& \sum_{j=1}^{N} a_{ij}(e^{S(t-t_k^i)}\hat{v}_i(t_k^i) - e^{S(t-t_{k'}^j)}\hat{v}_j(t_{k'}^j)) \\
&+ a_{i0}(e^{S(t-t_k^i)}\hat{v}_i(t_k^i) - e^{S(t-t_k^i)}v(t_k^i)), \\
\dot{d}_i(t) =& -\sigma_{1i}d_i(t) + \alpha_i\phi_i^\top(t)\phi_i(t),
\end{aligned} \tag{21}$$

where $\sigma_{1i} > 0$, and $d_i(t)$ is the adaptive parameter with $d_i(0) > 0$. The event-triggering strategy is expressed as

$$\begin{aligned}
t_{k+1}^i &= \inf\{t > t_k^i | (\psi_i(t)d_i(t) + 1)e_{\hat{v}i}^\top(t)e_{\hat{v}i}(t) > e^{-\gamma t}\}, \\
\dot{\psi}_i(t) &= -\sigma_{2i}\psi_i(t) + \beta_i d_i(t)e_{\hat{v}i}^\top(t)e_{\hat{v}i}(t),
\end{aligned} \tag{22}$$

where $\sigma_{2i} > 0$ and $\psi_i(0) > 0$. The rest of the parameters are designed the same as in Theorem 1.

Remark 6. The adaptive parameters $d_i(t)$ and $\psi_i(t)$ are designed based on σ-modification, which does not require any prior information about the bounds for the system disturbance. This modification adds damping to the ideal adaptive law in (5) and (6), so that the adaptive parameters will be bounded when the additive disturbance appears.

Now, we are ready to present the following theorem.

Theorem 3. *Consider the multi-agent system (20) and the exosystem (3). Suppose that Assumptions 1–4 hold. Under the control law (21) and the event-triggering scheme (22), there exists an open neighborhood $\mathcal{W}$ of $\omega = 0$, for any $\omega \in \mathcal{W}$ and any initial condition, the closed-loop system has the following properties.*
(1) All the variables of the closed-loop system are bounded for $t \ge 0$.
(2) $\lim_{t\to\infty} \|e_i(t)\| \le \|C_{\omega i}\| \frac{\sqrt{\kappa_i}}{\sqrt{\hat{\varrho}_1}}$, $i = 1,\ldots,N$.

Moreover, Zeno-behavior can be excluded for each agent.

Proof. Choose the same positive definite function $W(t)$ as in Theorem 1. The time derivative of $W(t)$ on each interval $[t_m, t_{m+1})$ can be given by

$$\begin{aligned}
\dot{W}(t) \le& -\sum_{i=1}^{N} \Big(\hat{\varrho}_1 x_{ci}^\top(t)x_{ci}(t) + \varrho_3\bar{v}_i^\top(t)\bar{v}_i(t) \\
&+ \frac{\sigma_{1i}}{4\alpha_i}\tilde{d}_i^2(t) + \frac{\sigma_{2i}}{2\beta_i}\tilde{\psi}_i^2(t) - \kappa_i\Big),
\end{aligned} \tag{23}$$

where $\kappa_i = \frac{\sigma_{1i}}{4\alpha_i}a_1^2 + \frac{\sigma_{2i}}{2\beta_i}a_2^2 + \frac{2p_2}{\epsilon}\|E_{di}\|^2\bar{\vartheta}_i^2$, $a_1 > \frac{4}{\lambda_{\min}(\mathcal{H}^\top\mathcal{H})} \times (2\|S\|\|\mathcal{H}\| + \frac{2p_2\|G_2F\|^2}{\epsilon} + \varrho_2)$, $\varrho_2 \ge \frac{8}{\gamma}N^2\alpha_{\max}\|\mathcal{H}\|^2 + \varrho_3$, $a_2 \ge 4(N+1)^2$, $a_3 \ge \frac{a_1}{2}\|\mathcal{H}\|^2 + 1 + \frac{8}{\gamma}N^2\alpha_{\max}\|\mathcal{H}\|^2$, and $\varrho_3 > 0$. Consequently $\dot{W}(t) < 0$, $t \in [t_m, t_{m+1})$ whenever the trajectory of the closed-loop system is outside of the set

$$\begin{aligned}
\Omega =& \{(x_{ci}(t), \bar{v}_i(t), \tilde{d}_i(t), \tilde{\psi}_i(t)) : \frac{\sigma_{1i}}{4\alpha_i}\tilde{d}_i^2(t) + \frac{\sigma_{2i}}{2\beta_i}\tilde{\psi}_i^2(t) \\
&+ \hat{\varrho}_1 x_{ci}^\top(t)x_{ci}(t) + \varrho_3\bar{v}_i^\top(t)\bar{v}_i(t) \le \kappa_i\}.
\end{aligned} \tag{24}$$

Similar to the proof in Theorem 1, one can also obtain that no agent will exhibit Zeno behavior at any time. Therefore, one can obtain that $\lim_{t\to\infty}\|\tilde{x}_i(t)\| \le \frac{\sqrt{\kappa_i}}{\sqrt{\hat{\varrho}_1}}$, $\lim_{t\to\infty}\|\tilde{z}_i(t)\| \le \frac{\sqrt{\kappa_i}}{\sqrt{\hat{\varrho}_1}}$, $\lim_{t\to\infty}\|\bar{v}_i(t)\| \le \frac{\sqrt{\kappa_i}}{\sqrt{\varrho_3}}$, $\lim_{t\to\infty}\|\tilde{d}_i(t)\| \le \frac{\sqrt{4\alpha_i\kappa_i}}{\sqrt{\sigma_{1i}}}$ and $\lim_{t\to\infty}\|\tilde{\psi}_i(t)\| \le \frac{\sqrt{2\beta_i\kappa_i}}{\sqrt{\sigma_{2i}}}$. The regulated output $e_i(t)$ satisfies $\lim_{t\to\infty}\|e_i(t)\| \le \lim_{t\to\infty}\|C_{\omega i}\|\|\tilde{x}_i(t)\| \le \|C_{\omega i}\|\frac{\sqrt{\kappa_i}}{\sqrt{\hat{\varrho}_1}}$. The proof of Theorem 3 is completed. □

4. An example

In this section, simulation results are provided to illustrate the effectiveness of the proposed controllers. Consider a group of 4 agents borrowed from Su et al. (2013) with $A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $E_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & i \end{bmatrix}$, $C_i = [\,1\ 0\,]$, $F = [\,-1\ -1\ 0\,]$, $\Delta A_i = \begin{bmatrix} 0 & 0 \\ \delta_{1i} & \delta_{2i} \end{bmatrix}$, and $S = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, where $\delta_{ji} = 0.05 \times j \times i$, $j = 1,2$, $i = 1,2,3,4$. Since (G_1, G_2) incorporates a 1-copy internal

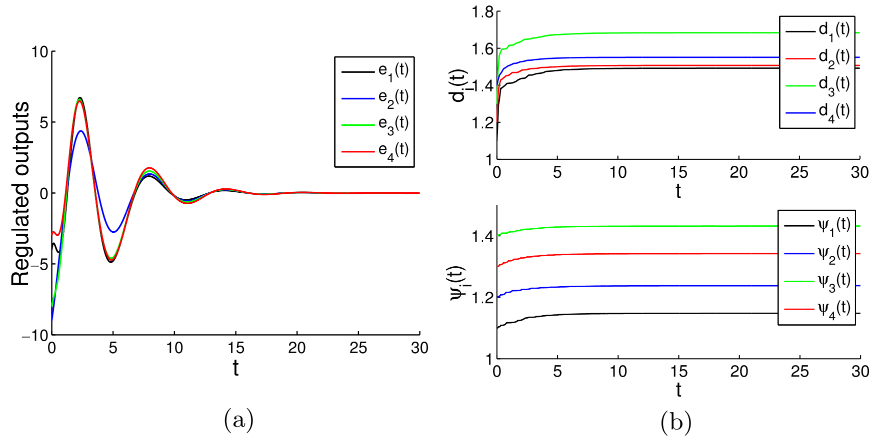

Fig. 1. (a) The regulated outputs $e_i(t)$, $i = 1, \dots, 4$. (b) Adaptive parameters $d_i(t)$ and $\psi_i(t)$, $i = 1, \dots, 4$ under controller (5).

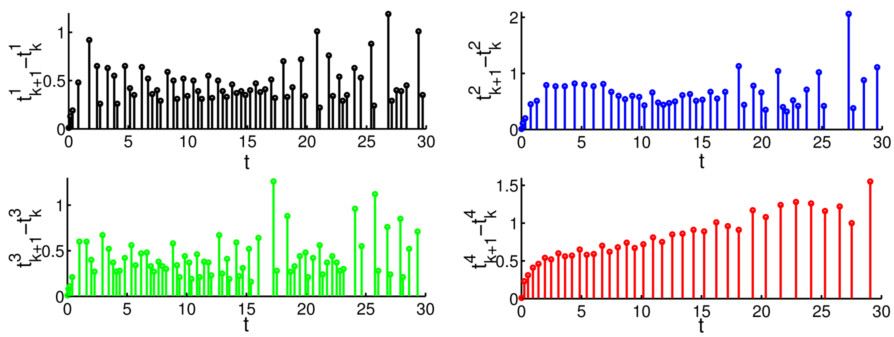

Fig. 2. Triggering instants and interval times.

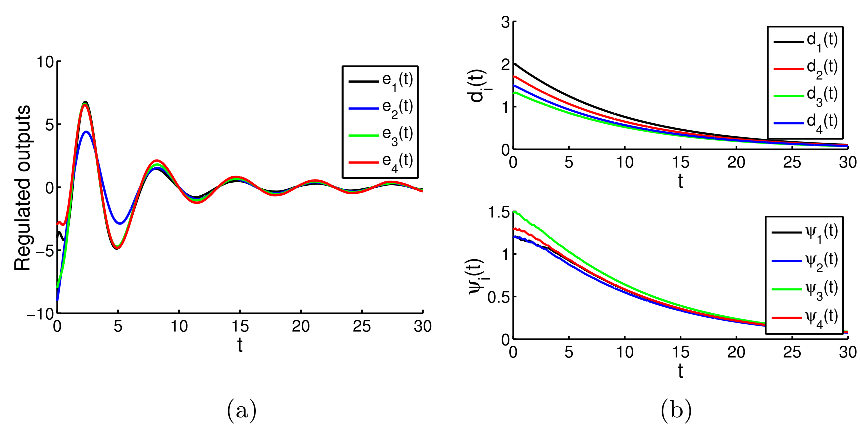

Fig. 3. (a) The regulated outputs $e_i(t)$, $i = 1, \dots, 4$. (b) Adaptive parameters $d_i(t)$ and $\psi_i(t)$, $i = 1, \dots, 4$ under controller (21).

model of S, one has $G_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$, $G_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. The communication topology of the multi-agent system is described by $\mathcal{H} = [\, 3, -1\,, -1, 0; -1, 2, -1, 0; -1, -1, 3, -1; 0, 0, -1, 1 \,]$.

Case 1. Consider the multi-agent system (1) without additive disturbances. Choosing $\varrho_1 = 5$, $\gamma = 0.5$, $\mu_i = 5$, $\alpha_i = 0.05$, and $\beta_i = 0.1$, one can obtain that $K_{xi} = [\, -10.1123 \;\; -4.7552 \,]$ and $K_{zi} = [\, -3.5355 \;\; -1.1213 \;\; -9.5557 \,]$, $i = 1, \dots, 4$. Under the controller (5) and the event-triggering scheme (6), simulation results are shown in Figs. 1–2.

The regulated outputs $e_i(t)$, $i = 1, \dots, 4$ are shown in Fig. 1(a). It can be observed that the regulated outputs converge to zero asymptotically, which means that under the proposed control strategy (5), the multi-agent system achieves robust cooperative output regulation asymptotically. The adaptive parameters $d_i(t)$ and $\psi_i(t)$, $i = 1, \dots, 4$ are depicted in Fig. 1(b), from which it can be observed that $d_i(t)$ and $\psi_i(t)$ converge to some finite steady-state values asymptotically. The triggering instants are shown in Fig. 2. One can obtain that the average inter-event interval is 0.5597s. It should also be noted that by using the fixed period sampling control strategy with the sampling period of 0.5597s, robust cooperative output regulation cannot be achieved.

Case 2. Consider the multi-agent system (20) with bounded additive disturbances, where the system matrix $E_{di} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and the value of additive disturbance $\vartheta_i(t)$ is randomly extracted in the range $[-1, 1]$. Choosing $\varrho_1 = 5$, $\gamma = 0.5$, $\mu_i = 5$, $\alpha_i = 0.01$, $\beta_i = 0.5$, $\sigma_{1i} = 0.1$, and $\sigma_{2i} = 0.1$. Under the controller (21) and the event-triggering scheme (22), the trajectories of the regulated output $e_i(t)$ and adaptive parameters $d_i(t)$ and $\psi_i(t)$ are shown in

Fig. 3. It can be observed that the proposed controller could efficiently reduce the effect of additive disturbances, and the regulated output $e_i(t)$ converges to a bounded range asymptotically.

5. Conclusion

Two novel adaptive event-triggered control strategies are proposed for the uncertain linear multi-agent systems with additive disturbances. The proposed control strategies need no global knowledge such as the minimal nonzero eigenvalue of the Laplacian matrix, and require much less data transmission. One possible future research can be directed to cooperative control of heterogeneous multi-agent systems with unknown linear systems, or even nonlinear agent dynamics.

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